1. Write a proposition.

2. Construct a truth table for \((p \leftrightarrow q) \leftrightarrow (r \leftrightarrow q)\).

3. Let \(F(x, y)\) be the statement “\(x\) can fool \(y\),” where the universe of discourse consists of all the people in the world. Use quantifiers to express each of these statements.
   a)Everybody can fool Fred.
   b)Evelyn can fool everybody.
   c)Everybody can fool somebody.
   d)There is no one who can fool everybody.
   e)Everyone can be fooled by somebody.
   f)No one can fool both Fred and Jerry.
   h)There is exactly one person whom everybody can fool.
   i)No one can fool himself or herself.
   j)There is someone who can fool exactly one person besides himself or herself.

4. Write, in English, the contrapositive of the statement “If you take the subway, you have to buy a metrocard.”
5. Prove the following statement with either a membership table or using set identities.

\[(A - C) \cap (C - B) = \emptyset\]

6. What is \(P\{1\}\), where \(P\) means power set?

8. Can you write “A \(\cap\) B” using only A, B, \(\cup\), \(\neg\) (complement of set, e.g. \(\neg A\))

9. Define a function on the integers that is onto but not one-to-one.

10. Determine whether each of these functions is a bijection from \(\mathbb{R}\) to \(\mathbb{R}\). (\(\mathbb{R}\) is the set or reals)
    a) \(f(x) = -3x + 4\)
    b) \(f(x) = -3x^2 + 7\)
    c) \(f(x) = (x + 1)/(x + 2)\)
    d) \(f(x) = x^5 + 1\)

11. Prove or disprove that \(2^n - 1\) is prime for all integers \(n\). (Primes of the form \(2^n - 1\) are called Mersenne primes)

12. Prove that if \(n\) is an even, positive integer then \(7n + 4\) is even.
13. Prove that 5 divides n if and only if 5 divides $n^2$

14. Call an imaginary integer a complex number which consists of real and imaginary portions which are represented by integers. Is the set of imaginary numbers countable? Prove or disprove.

15. Prove the set of real numbers is not countable.

16. Prove by mathematical induction, that for all integer n where n > 1:

$$\sum_{i=1}^{n} \frac{1}{i^2} < 2 - \frac{1}{n}$$
17. Prove by mathematical induction, that for all integer $n$ where $n > 0$:

$$\sum_{i=1}^{n} i(i+1)(i+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

18. Prove by mathematical induction, for all positive integer $n$,

$$(2n)! \geq (n!)^2$$

19. How many ways can you split 20 people into 4 committees of 5, given that each person serves on exactly one committee?

20. How many ways you make 4 committees of 5 people out of 20 people if people can serve on more than one committee?
21. John always wears jeans. He either wears a long-sleeved shirt or a t-shirt and flannel jacket. John owns 5 pairs of jeans, 6 long-sleeved shirts, 12 t-shirts and 2 flannel jackets. How many ways can John get dressed.

22. You are the judge of a photo contest with 100 different submissions. There are 17 awards. 1 first prize, 1 second prize, 5 honorable mentions, and 10 “nice tries.” Each submission can get one award at most. How many ways can you distribute the prizes?

23. Suppose you have $100 in identical $1 bills. How many ways can you give all of this money to 10 different friends?

24. What is the coefficient of $x^{10}$ in $(2 + x)^{100}$?

BONUS: Is the set of real numbers with decimal representations consisting of all 1s (e.g. 1.1, 1111.1, or .1111111) countable. Prove your answer.