1. Construct a finite-state machine that models a vending machine accepting only quarters that gives a container of orange juice when 50 cents has been deposited, followed by a button being pushed. (The possible inputs are quarters and the button, and the possible outputs are nothing, orange juice, and a quarter. The machine returns any extra quarters.)

2. Construct a finite-state machine with output that produces a 1 if and only if the last 3 input bits read are 0s.

3. Determine the output for each input string 10111. using the state table shown below:

<table>
<thead>
<tr>
<th>$f$</th>
<th>$G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>input:</td>
<td>input:</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$s_0$</td>
<td>$s_1$</td>
</tr>
<tr>
<td>$s_1$</td>
<td>$s_2$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>$s_1$</td>
</tr>
<tr>
<td>$s_3$</td>
<td>$s_3$</td>
</tr>
</tbody>
</table>
4. Find the set recognized by the following deterministic finite-state machine.

5. Which strings are recognized by the following finite-state automaton?

6. Prove that \( x^3 + 7x + 2 \) is \( \Omega(x^3) \).

7. Draw two non-isomorphic spanning trees of the complete bi-partite graph \( K_{3,4} \).

8. Find the chromatic number for the graph below. Present a coloring in support of your answer.
9. Use the definition of big-oh to prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + (n - 1) \cdot n$ is $O(n^3)$.

10. Consider the relation $R$ on $\{w, x, y, z\}$ where $R = \{(w, w), (w, x), (x, w), (x, x), (x, z), (y, y), (z, z), (z, y)\}$. Determine whether the binary relation is: (1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive.

11. Suppose $A = \{2, 3, 6, 9, 10, 12, 14, 18, 20\}$ and $R$ is the partial order relation defined on $A$ where $xRy$ means $x$ is a divisor of $y$.
   (a) Draw the Hasse diagram for $R$.
   (b) Find all maximal elements.
   (c) Find all minimal elements.
   (d) Find lub($\{2, 9\}$).
   (e) Find lub($\{3, 10\}$).
   (f) Find glb($\{14, 10\}$).

12. Draw a simple graph with degree sequence 2, 3, 4, 4, 4 or show that one cannot exist.

13. Draw a simple graph with a Hamiltonian Path but no Hamiltonian Circuit.
14. A simple graph is said to be a cubic graph if it is simple and every vertex has degree 3. Draw a cubic graph with 6 vertices that is not isomorphic to $K_{3,3}$.

15. If $T$ is a full binary tree with 50 leaves then state and argue what its minimum height would be.

16. Suppose $T$ is a full $m$-ary tree with $l$ leaves. Prove that $T$ has $(l - 1)/(m - 1)$ internal vertices.

17. If $G$ is a connected graph with 12 regions and 20 edges, then state $G$ has _______ vertices. Explain your answer.

18. Show a graph with 4 vertices that is not planar and argue that none can exist.