9.5 #10: Can someone cross all the bridges shown in this map exactly once and return to the starting point?

I’ll skip the picture and simply put in the graph you get when you turn the land masses into vertices and the bridges into edges.

The degrees of each node are: 2, 4, 4, and 2. Since every vertex has an even degree you know that there is a Euler circuit (which is what the question is really asking). One possible Euler circuit: A – B – C – B – D – C – A.

9.5 #16 Is there a Euler circuit in this graph? If not, is there a Euler path?

To tell if there is a Euler circuit, we check whether every vertex has an even degree. In this graph, b and c have degrees of 3 so we immediately know that there is no Euler circuit. However, since there are exactly two vertices with an odd degree there is an Euler path. One possible path is: b – c – i – b – a – i – g – d – f – e – d – a – h – i – d – c. It is worth noting that the path must start at one of the odd vertices and end at the other.
9.7 #6 Is this graph planar? If so, draw it as planar.

We can tell this graph is planar because it does not have $K_5$ or $K_{3,3}$ as a subgraph. It can be drawn like this:

9.8 #8 Find the following graph’s chromatic number

Using the method I talked about in class, we can suppose the chromatic number will be equal to the largest complete graph found as a subgraph of the overall graph. The biggest complete subgraph is $K_3$ (a, e, c or b, f, d) so we can suppose the graph can be colored in three colors. And it can.

$$
\begin{align*}
  a &= \text{red} & b &= \text{green} \\
  e &= \text{blue} & f &= \text{red} \\
  c &= \text{green} & d &= \text{blue}
\end{align*}
$$
9.8 #10 Find the following graph’s chromatic number

Doing the same as above we look for the biggest complete subgraph. We find i, h, b, and c forming a complete graph of size 4. There is no complete graph of size 5 so we assume the chromatic number is 4. It is:

\[ a = \text{red} \quad b = \text{green} \quad c = \text{red} \quad d = \text{green} \]
\[ e = \text{red} \quad f = \text{green} \quad g = \text{blue} \quad h = \text{black} \]
\[ i = \text{green} \]

10.1 #16 Which complete bipartite graphs $K_{m,n}$ where $m$ and $n$ are positive integers, are trees.

A tree is a graph with no cycles (among other traits). So we have to figure out which complete bipartite graphs have cycles. Obvious $K_{1,1}$ does not, $K_{2,1}$ does not. But $K_{2,2}$ does. And when you consider any bipartite graphs with more vertices, you can be sure that $K_{2,2}$ will be a subgraph. So, from this we can draw the conclusion that anytime $m$ and $n$ are both greater than 2, there will be cycle. So the answer is any bipartite graph where $m$ or $n$ is equal to 1.

10.1 #18 How many vertices does a full 5-ary tree with 100 internal vertices have?

This question simply asks you to plug these values into the equation from theorem 4ii from section 10.1 in the book. The part we are interested in reads, “A full m-ary tree with I internal vertices has \( n = mi + 1 \) vertices…” So know two of those values and can find the third which is the total number of vertices.

\[ m = 5 \]
\[ i = 100 \]
\[ n = 5(100) + 1 \]
\[ n = 501 \]