Solution: Let $P(n)$ denote "$n > 4$" and $Q(n)$ denote "$n^2 < 2^n$." The statement "For all positive integers $n$, if $n$ is greater than 4, then $n^2$ is less than $2^n$" can be represented by $\forall n (P(n) \rightarrow Q(n))$, where the domain consists of all positive integers. We are assuming that $\forall n (P(n) \rightarrow Q(n))$ is true. Note that $P(100)$ is true because $100 > 4$. It follows by universal modus ponens that $Q(n)$ is true, namely that $100^2 < 2^{100}$.

Another useful combination of a rule of inference from propositional logic and a rule of inference for quantified statements is universal modus tollens. Universal modus tollens combines universal instantiation and modus tollens and can be expressed in the following way:

\[
\forall x (P(x) \rightarrow Q(x)) \\
\neg Q(a), \text{ where } a \text{ is a particular element in the domain} \\
\therefore \neg P(a)
\]

We leave the verification of universal modus tollens to the reader (see Exercise 25). Exercise 26 develops additional combinations of rules of inference in propositional logic and quantified statements.

Exercises

1. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

   If Socrates is human, then Socrates is mortal.
   Socrates is human.
   \therefore Socrates is mortal.

2. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

   If George does not have eight legs, then he is not an insect.
   George is an insect.
   \therefore George has eight legs.

3. What rule of inference is used in each of these arguments?

   a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
   b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
   e) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.
   d) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.
   e) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

4. What rule of inference is used in each of these arguments?

   a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
   b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
   c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
   d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.
   e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

5. Use rules of inference to show that the hypotheses “Randy works hard,” “If Randy works hard, then he is a dull boy,” and “If Randy is a dull boy, then he will not get the job” imply the conclusion “Randy will not get the job.”

6. Use rules of inference to show that the hypotheses “If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,” “If the sailing race is held, then the trophy will be awarded,” and “The trophy was not awarded” imply the conclusion “It rained.”

7. What rules of inference are used in this famous argument? “All men are mortal. Socrates is a man. Therefore, Socrates is mortal.”

8. What rules of inference are used in this argument? “No man is an island. Manhattan is an island. Therefore, Manhattan is not a man.”
9. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.
   a) “If I take the day off, it either rains or snows.” “I took Tuesday off or I took Thursday off.” “It was sunny on Tuesday.” “It did not snow on Thursday.”
   b) “If I eat spicy foods, then I have strange dreams.” “I have strange dreams if there is thunder while I sleep.” “I did not have strange dreams.”
   c) “I am either clever or lucky.” “I am not lucky.” “If I am lucky, then I will win the lottery.”
   d) “Every computer science major has a personal computer.” “Ralph does not have a personal computer.” “Ann has a personal computer.”
   e) “What is good for corporations is good for the United States.” “What is good for the United States is good for you.” “What is good for corporations is for you to buy lots of stuff.”
   f) “All rodents gnaw their food.” “Mice are rodents.” “Rabbits do not gnaw their food.” “Bats are not rodents.”

10. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.
   a) “If I play hockey, then I am sore the next day.” “I use the whirlpool if I am sore.” “I did not use the whirlpool.”
   b) “If I work, it is either sunny or partly sunny.” “I worked last Monday or I worked last Friday.” “It was not sunny on Tuesday.” “It was not partly sunny on Friday.”
   c) “All insects have six legs.” “Dragonflies are insects.” “Spiders do not have six legs.” “Spiders eat dragonflies.”
   d) “Every student has an Internet account.” “Homer does not have an Internet account.” “Maggie has an Internet account.”
   e) “All foods that are healthy to eat do not taste good.” “Tofu is healthy to eat.” “You only eat what tastes good.” “You do not eat tofu.” “Cheeseburgers are not healthy to eat.”
   f) “I am either dreaming or hallucinating.” “I am not dreaming.” “If I am hallucinating, I see elephants running down the road.”

11. Show that the argument form with premises \( p_1, p_2, \ldots, p_n \) and conclusion \( q \rightarrow r \) is valid if the argument form with premises \( p_1, p_2, \ldots, p_n, q \), and conclusion \( r \) is valid.

12. Show that the argument form with premises \( (p \land q) \rightarrow (r \lor s), q \rightarrow (u \land t), u \rightarrow p, \) and \( \neg v \) and conclusion \( q \rightarrow r \) is valid by first using Exercise 11 and then using rules of inference from Table 1.

13. For each of these arguments, explain which rules of inference are used for each step.
   a) “Doug, a student in this class, knows how to write programs in JAVa. Everyone who knows how to write programs in JAVa can get a high-paying job. Therefore, someone in this class can get a high-paying job.”
   b) “Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution.”
   c) “Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program.”
   d) “Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean.”

14. For each of these arguments, explain which rules of inference are used for each step.
   a) “Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket.”
   b) “Each of five roommates, Melissa, Aaron, Ralph, Venus, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year.”
   c) “All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners.”
   d) “There is someone in this class who has been to France. Everyone who goes to France visits the Louvre. Therefore, someone in this class has visited the Louvre.”

15. For each of these arguments determine whether the argument is correct or incorrect and explain why.
   a) All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.
   b) Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major.
   c) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.
   d) Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

16. For each of these arguments determine whether the argument is correct or incorrect and explain why.
   a) Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.
   b) A convertible car is fun to drive. Isaac’s car is not a convertible. Therefore, Isaac’s car is not fun to drive.
   c) Quincy likes all action movies. Quincy likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie.
d) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.

17. What is wrong with this argument? Let \( H(x) \) be "\( x \) is happy." Given the premise \( \exists x H(x) \), we conclude that \( H(Lola) \). Therefore, Lola is happy.

18. What is wrong with this argument? Let \( S(x, y) \) be "\( x \) is shorter than \( y \)." Given the premise \( \exists x S(x, \text{Max}) \), it follows that \( S(\text{Max}, \text{Max}) \). Then by existential generalization it follows that \( \exists x S(x, x) \), so that someone is shorter than himself.

19. Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?

   a) If \( n \) is a real number such that \( n > 1 \), then \( n^2 > 1 \). Suppose that \( n^2 > 1 \). Then \( n > 1 \).
   b) If \( n \) is a real number with \( n > 3 \), then \( n^2 > 9 \). Suppose that \( n^2 \leq 9 \). Then \( n \leq 3 \).
   c) If \( n \) is a real number with \( n > 2 \), then \( n^2 > 4 \). Suppose that \( n \leq 2 \). Then \( n^2 \leq 4 \).

20. Determine whether these are valid arguments.

   a) If \( x \) is a positive real number, then \( x^2 \) is a positive real number. Therefore, if \( a^2 \) is positive, where \( a \) is a real number, then \( a \) is a positive real number.
   b) If \( x^2 \neq 0 \), where \( x \) is a real number, then \( x \neq 0 \). Let \( a \) be a real number with \( a^2 \neq 0 \). Then \( a \neq 0 \).

21. Which rules of inference are used to establish the conclusion of Lewis Carroll’s argument described in Example 26 of Section 1.3?

22. Which rules of inference are used to establish the conclusion of Lewis Carroll’s argument described in Example 27 of Section 1.3?

23. Identify the error or errors in this argument that supposedly shows that if \( \exists x P(x) \land \exists x Q(x) \) is true then \( \exists x (P(x) \lor Q(x)) \) is true.

   1. \( \exists x P(x) \land \exists x Q(x) \)
   2. \( \exists x P(x) \)
   3. \( P(c) \)
   4. \( \exists x Q(x) \)
   5. \( Q(c) \)
   6. \( P(c) \lor Q(c) \)
   7. \( \exists x (P(x) \lor Q(x)) \)

24. Identify the error or errors in this argument that supposedly shows that if \( \forall x (P(x) \lor Q(x)) \) is true then \( \forall x P(x) \lor \forall x Q(x) \) is true.

   1. \( \forall x (P(x) \lor Q(x)) \)
   2. \( P(c) \lor Q(c) \)
   3. \( P(c) \)
   4. \( \forall x P(x) \)
   5. \( Q(c) \)
   6. \( \forall x Q(x) \)
   7. \( \forall x (P(x) \lor \forall x Q(x)) \)

25. Justify the rule of universal modus tollens by showing that the premises \( \forall x (P(x) \rightarrow Q(x)) \) and \( \neg Q(a) \) for a particular element \( a \) in the domain, imply \( \neg P(a) \).

26. Justify the rule of universal transitivity, which states that if \( \forall x (P(x) \rightarrow Q(x)) \) and \( \forall x (Q(x) \rightarrow R(x)) \) are true, then \( \forall x (P(x) \rightarrow R(x)) \) is true, where the domains of all quantifiers are the same.

27. Use rules of inference to show that if \( \forall x (P(x) \rightarrow (Q(x) \land S(x))) \) and \( \forall x (P(x) \land R(x)) \) are true, then \( \forall x (R(x) \rightarrow S(x)) \) is true.

28. Use rules of inference to show that if \( \forall x (P(x) \lor Q(x)) \) and \( \forall x ((\neg P(x) \land Q(x)) \rightarrow R(x)) \) are true, then \( \forall x (\neg R(x) \rightarrow P(x)) \) is also true, where the domains of all quantifiers are the same.

29. Use rules of inference to show that if \( \forall x (P(x) \lor Q(x)), \forall x (\neg Q(x) \lor S(x)), \forall x (R(x) \rightarrow \neg S(x)), \) and \( \exists x \neg P(x) \) are true, then \( \exists x \neg R(x) \) is true.

30. Use resolution to show the hypotheses "Allen is a bad boy or Hillary is a good girl" and "Allen is a good boy or David is happy" imply the conclusion "Hillary is a good girl or David is happy."

31. Use resolution to show that the hypotheses "It is not raining or Yvette has her umbrella," "Yvette does not have her umbrella or she does not get wet," and "It is raining or Yvette does not get wet" imply that "Yvette does not get wet."

32. Show that the equivalence \( p \land \neg p \equiv F \) can be derived using resolution together with the fact that a conditional statement with a false hypothesis is true. [Hint: Let \( q = r = F \) in resolution.]

33. Use resolution to show that the compound proposition \( (p \lor q) \land (\neg p \lor q) \land (p \lor \neg q) \land (\neg p \lor \neg q) \) is not satisfiable.

*34. The Logic Problem, taken from WFF’N PROOF, The Game of Logic, has these two assumptions:
1. "Logic is difficult or not many students like logic."
2. "If mathematics is easy, then logic is not difficult."

By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions:

a) That mathematics is not easy, if many students like logic.

b) That not many students like logic, if mathematics is not easy.

c) That mathematics is not easy or logic is difficult.

d) That logic is not difficult or mathematics is not easy.

e) That if not many students like logic, then either mathematics is not easy or logic is not difficult.

*35. Determine whether this argument, taken from Kalish and Montague [KalMo64], is valid.

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.
The $3x + 1$ conjecture has an interesting history and has attracted the attention of mathematicians since the 1950s. The conjecture has been raised many times and goes by many other names, including the Collatz problem, Hasse's algorithm, Ulam's problem, the Syracuse problem, and Kakutani's problem. Many mathematicians have been diverted from their work to spend time attacking this conjecture. This led to the joke that this problem was part of a conspiracy to slow down American mathematical research. See the article by Jeffrey Lagarias [La85] for a fascinating discussion of this problem and the results that have been found by mathematicians attacking it.

In Chapter 3 we will describe additional open questions about prime numbers. Students already familiar with the basic notions about primes might want to explore Section 3.4, where these open questions are discussed. We will mention other important open questions throughout the book.

**Additional Proof Methods**

In this chapter we introduced the basic methods used in proofs. We also described how to leverage these methods to prove a variety of results. We will use these proof methods in Chapters 2 and 3 to prove results about sets, functions, algorithms, and number theory. Among the theorems we will prove is the famous halting theorem which states that there is a problem that cannot be solved using any procedure. However, there are many important proof methods besides those we have covered. We will introduce some of these methods later in this book. In particular, in Section 4.1 we will discuss mathematical induction, which is an extremely useful method for proving statements of the form $\forall n P(n)$, where the domain consists of all positive integers. In Section 4.3 we will introduce structural induction, which can be used to prove results about recursively defined sets. We will use the Cantor diagonalization method, which can be used to prove results about the size of infinite sets, in Section 2.4. In Chapter 5 we will introduce the notion of combinatorial proofs, which can be used to prove results by counting arguments. The reader should note that entire books have been devoted to the activities discussed in this section, including many excellent works by George Pólya ([Po61], [Po71], [Po90]).

Finally, note that we have not given a procedure that can be used for proving theorems in mathematics. It is a deep theorem of mathematical logic that there is no such procedure.

**Exercises**

1. Prove that $n^2 + 1 \geq 2^n$ when $n$ is a positive integer with $1 \leq n \leq 5$.
2. Prove that there are no positive perfect cubes less 1000 that are the sum of the cubes of two positive integers.
3. Prove that if $x$ and $y$ are real numbers, then $\max(x, y) + \min(x, y) = x + y$. \textit{[Hint: Use a proof by cases, with the two cases corresponding to $x \geq y$ and $x < y$, respectively.]}  
4. Use a proof by cases to show that $\min(a, \min(b, c)) = \min(\min(a, b), c)$ whenever $a$, $b$, and $c$ are real numbers.
5. Prove the triangle inequality, which states that if $x$ and $y$ are real numbers, then $|x| + |y| \geq |x + y|$ (where $|x|$ represents the absolute value of $x$, which equals $x$ if $x \geq 0$ and equals $-x$ if $x < 0$).
6. Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or nonconstructive?
7. Prove that there are 100 consecutive positive integers that are not perfect squares. Is your proof constructive or nonconstructive?
8. Prove that either $2 \cdot 10^{500} + 15$ or $2 \cdot 10^{500} + 16$ is not a perfect square. Is your proof constructive or nonconstructive?
9. Prove that there exists a pair of consecutive integers such that one of these integers is a perfect square and the other is a perfect cube.
10. Show that the product of two of the numbers $65^{1000} - 8^{2001} + 3^{177}, 79^{1212} - 9^{2399} + 2^{2001},$ and $24^{4993} - 5^{192} +$
7^{177} is nonnegative. Is your proof constructive or non-constructive? [Hint: Do not try to evaluate these numbers!]

11. Prove or disprove that there is a rational number \( x \) and an irrational number \( y \) such that \( x^y \) is irrational.

12. Prove or disprove that if \( a \) and \( b \) are rational numbers, then \( a^b \) is also rational.

13. Show that each of these statements can be used to express the fact that there is a unique element \( x \) such that \( P(x) \) is true. [Note that we can also write this statement as \( \exists! x P(x) \).]
   a) \( \exists x \forall y (P(y) \iff x = y) \)
   b) \( \exists x P(x) \land \forall x \forall y (P(x) \land P(y) \rightarrow x = y) \)
   c) \( \exists x (P(x) \land \forall y (P(y) \rightarrow x = y)) \)

14. Show that if \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \), then there is a unique solution of the equation \( ax + b = c \).

15. Suppose that \( a \) and \( b \) are odd integers with \( a \neq b \). Show there is a unique integer \( c \) such that \( |a - c| = |b - c| \).

16. Show that if \( r \) is an irrational number, there is a unique integer \( n \) such that the distance between \( r \) and \( n \) is less than \( 1/2 \).

17. Show that if \( n \) is an odd integer, then there is a unique integer \( k \) such that \( n \) is the sum of \( k - 2 \) and \( k + 3 \).

18. Prove that given a real number \( x \) there exist unique numbers \( n \) and \( \epsilon \) such that \( x = n + \epsilon \), \( n \) is an integer, and \( 0 \leq \epsilon < 1 \).

19. Prove that given a real number \( x \) there exist unique numbers \( n \) and \( \epsilon \) such that \( x = n - \epsilon \), \( n \) is an integer, and \( 0 \leq \epsilon < 1 \).

20. Use forward reasoning to show that if \( x \) is a nonzero real number, then \( x^2 + 1/x^2 \geq 2 \). [Hint: Start with the inequality \( (x - 1/x)^2 \geq 0 \) which holds for all nonzero real numbers \( x \).]

21. The harmonic mean of two real numbers \( x \) and \( y \) equals \( 2xy/(x+y) \). By computing the harmonic and geometric means of different pairs of positive real numbers, formulate a conjecture about their relative sizes and prove your conjecture.

22. The quadratic mean of two real numbers \( x \) and \( y \) equals \( \sqrt{(x^2 + y^2)/2} \). By computing the arithmetic and quadratic means of different pairs of positive real numbers, formulate a conjecture about their relative sizes and prove your conjecture.

*23. Write the numbers 1, 2, \ldots, 2n on a blackboard, where \( n \) is an odd integer. Pick any two of the numbers, \( j \) and \( k \), write \( |j - k| \) on the board and erase \( j \) and \( k \). Continue this process until only one integer is written on the board. Prove that this integer must be odd.

*24. Suppose that five ones and four zeros are arranged around a circle. Between any two equal bits you insert a 0 and between any two unequal bits you insert a 1 to produce nine new bits. Then you erase the nine original bits. Show that when you iterate this procedure, you can never get nine zeros. [Hint: Work backward, assuming that you did end up with nine zeros.]

25. Formulate a conjecture about the decimal digits that appear as the final decimal digit of the fourth power of an integer. Prove your conjecture using a proof by cases.

26. Formulate a conjecture about the final two decimal digits of the square of an integer. Prove your conjecture using a proof by cases.

27. Prove that there is no positive integer \( n \) such that \( n^2 + n^3 = 100 \).

28. Prove that there are no solutions in integers \( x \) and \( y \) to the equation \( 2x^2 + 5y^2 = 14 \).

29. Prove that there are no solutions in positive integers \( x \) and \( y \) to the equation \( x^4 + y^4 = 625 \).

30. Prove that there are infinitely many solutions in positive integers \( x, y, \) and \( z \) to the equation \( x^2 + y^2 = z^2 \). [Hint: Let \( x = m^2 - n^2, y = 2mn, \) and \( z = m^2 + n^2 \), where \( m \) and \( n \) are integers.]

31. Adapt the proof in Example 4 in Section 1.6 to prove that if \( n = abc \), where \( a, b, \) and \( c \) are positive integers, then \( a \leq \sqrt[3]{n}, b \leq \sqrt[3]{n}, \) or \( c \leq \sqrt[3]{n} \).

32. Prove that \( \sqrt{2} \) is irrational.

33. Prove that between every two rational numbers there is an irrational number.

34. Prove that between every rational number and every irrational number there is an irrational number.

*35. Let \( S = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n \), where \( x_1, x_2, \ldots, x_n \) and \( y_1, y_2, \ldots, y_n \) are orderings of two different sequences of positive real numbers, each containing \( n \) elements.
   a) Show that \( S \) takes its maximum value over all orderings of the two sequences when both sequences are sorted (so that the elements in each sequence are in nondecreasing order).
   b) Show that \( S \) takes its minimum value over all orderings of the two sequences when one sequence is sorted into nondecreasing order and the other is sorted into nonincreasing order.

36. Prove or disprove that if you have an 8-gallon jug of water and two empty jugs with capacities of 5 gallons and 3 gallons, respectively, then you can measure 4 gallons by successively pouring some of or all of the water in a jug into another jug.

37. Verify the 3x + 1 conjecture for these integers.
   a) 6   b) 7   c) 17   d) 21

38. Verify the 3x + 1 conjecture for these integers.
   a) 16   b) 11   c) 35   d) 113

39. Prove or disprove that you can use dominoes to tile the standard checkerboard with two adjacent corners removed (that is, corners that are not opposite).

40. Prove or disprove that you can use dominoes to tile a standard checkerboard with all four corners removed.

41. Prove that you can use dominoes to tile a rectangular checkerboard with an even number of squares.

42. Prove or disprove that you can use dominoes to tile a 5 \times 5 checkerboard with three corners removed.
43. Use a proof by exhaustion to show that a tiling using dominoes of a 4 x 4 checkerboard with opposite corners removed does not exist. [Hint: First show that you can assume that the squares in the upper left and lower right corners are removed. Number the squares of the original checkerboard from 1 to 16, starting in the first row, moving right in this row, then starting in the leftmost square in the second row and moving right, and so on. Remove squares 1 and 16. To begin the proof, note that square 2 is covered either by a domino laid horizontally, which covers squares 2 and 3, or vertically, which covers squares 2 and 6. Consider each of these cases separately, and work through all the subcases that arise.]

44. Prove that when a white square and a black square are removed from an 8 x 8 checkerboard (colored as in the text) you can tile the remaining squares of the checkerboard using dominoes. [Hint: Show that when one black and one white square are removed, each part of the partition of the remaining cells formed by inserting the barriers shown in the figure can be covered by dominoes.]

45. Show that by removing two white squares and two black squares from an 8 x 8 checkerboard (colored as in the text) you can make it impossible to tile the remaining squares using dominoes.

46. Find all squares, if they exist, on an 8 x 8 checkerboard so that the board obtained by removing one of these squares can be tiled using straight trominoes. [Hint: First use arguments based on coloring and rotations to eliminate as many squares as possible from consideration.]

47. a) Draw each of the five different tetrominoes, where a tetromino is a polyomino consisting of four squares.

b) For each of the five different tetrominoes, prove or disprove that you can tile a standard checkerboard using these tetrominoes.

48. Prove or disprove that you can tile a 10 x 10 checkerboard using straight tetrominoes.

Key Terms and Results

TERMS

proposition: a statement that is true or false
propositional variable: a variable that represents a proposition
truth value: true or false
\( \neg p \) (negation of \( p \)): the proposition with truth value opposite to the truth value of \( p \)
logical operators: operators used to combine propositions
compound proposition: a proposition constructed by combining propositions using logical operators
truth table: a table displaying the truth values of propositions
\( p \lor q \) (disjunction of \( p \) and \( q \)): the proposition "\( p \) or \( q \)" which is true if and only if at least one of \( p \) and \( q \) is true
\( p \land q \) (conjunction of \( p \) and \( q \)): the proposition "\( p \) and \( q \)" which is true if and only if both \( p \) and \( q \) are true
\( p \oplus q \) (exclusive or of \( p \) and \( q \)): the proposition "\( p \) XOR \( q \)" which is true when exactly one of \( p \) and \( q \) is true
\( p \rightarrow q \) (\( p \) implies \( q \)): the proposition "if \( p \), then \( q \)" which is false if and only if \( p \) is true and \( q \) is false
converse of \( p \rightarrow q \): the conditional statement \( q \rightarrow p \)
contrapositive of \( p \rightarrow q \): the conditional statement \( \neg q \rightarrow \neg p \)

inverse of \( p \rightarrow q \): the conditional statement \( \neg p \rightarrow \neg q \)
\( p \leftrightarrow q \) (biconditional): the proposition "\( p \) if and only if \( q \)" which is true if and only if \( p \) and \( q \) have the same truth value
bit: either a 0 or a 1
Boolean variable: a variable that has a value of 0 or 1
bit operation: an operation on a bit or bits
bit string: a list of bits
bitwise operations: operations on bit strings that operate on each bit in one string and the corresponding bit in the other string
tautology: a compound proposition that is always true
contradiction: a compound proposition that is always false
contingency: a compound proposition that is sometimes true and sometimes false
consistent compound propositions: compound propositions for which there is an assignment of truth values to the variables that makes all these propositions true
logically equivalent compound propositions: compound propositions that always have the same truth values