7.1 #12 Assume the population of Earth in 2002 was 6.2 billion and grows at a rate of 1.3% a year.

a) Set up a recurrence relation for population of world n years after 2002.

Since each year’s population is 1.3% greater than the previous year, you can write the recurrence as:

\[ P(k) = 1.013 \times P(k-1) \]
\[ P(0) = 6,200,000,000 \]

b) Find explicit formula for population of world n years after 2002.

Since you know that the population is going to grow by 1.3% each year, you know that the population will be multiplied by 1.013 each year. So you can write a solution to the world’s population.

\[ P(k) = 6,200,000,000 \times (1.013)^n \]

(c) What will the population be in 2022?

This is a simple plug in to the equation found above.

\[ P(20) = 6,200,000,000 \times (1.013)^{20} \]
\[ P(20) = 8027505221 \]

7.2 #12 Find solution to \( a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3} \) for \( n = 3, 4, 5, \ldots \) with \( a_0 = 3, a_1 = 6, a_2 = 0 \)

Suppose \( a_n = r^n \)

\[ r^n = 2r^{n-1} + r^{n-2} - 2r^{n-3} \]
Divide by \( r^{n-3} \)

\[ r^3 = 2r^2 + r - 2 \]
\[ r^3 - 2r^2 - r + 2 = 0 \]
\[ r^2 (r-2) - 1(r-2) = 0 \]
\[ (r^2 -1)(r-2) = 0 \]
\[ r = 2, -1, 1 \]

So we have roots of 2, 1, and -1.

We plug them into the general form equation.

\[ a_n = \alpha_1 (2)^n + \alpha_2 (-1)^n + \alpha_3 (1)^n \]

Plugging in the base cases gives us the following equations.
\[ a_0 = 3 = \alpha_1 + \alpha_2 + \alpha_3 \]
\[ a_1 = 6 = 2\alpha_1 - \alpha_2 + \alpha_3 \]
\[ a_2 = 0 = 4\alpha_1 + \alpha_2 + \alpha_3 \]

From here you can use any method you like to solve this system of equations. A TI-89 is the easiest. =) But you don’t have to worry about such a large problem on a quiz. It’s unlikely that you’d get more than a two term recurrence relation.

You end up with:
\[ \alpha_1 = -1 \]
\[ \alpha_2 = -2 \]
\[ \alpha_3 = 6 \]

Which gives you the final answer:
\[ a_n = -1(2^n) - 2(-1)^n + 6(1)^n \]

7.2 #22 What is the general form of the solutions of a linear homogeneous recurrence if its characteristic equation has roots of -1, -1, -1, 2, 2, 5, 5, 7?

This question basically tests if you know how to handle roots of multiplicity of more than one (multiplicity means the number of times a number appears as a root). In this case -1 has a multiplicity of three, 2 has a multiplicity of two, and 5 has a multiplicity of two.

The general form of each individual term of the general equation is this: \( \left( \sum_{i=0}^{b} \alpha_i n^i \right) r^n \)

Suppose an equation has roots of 1, 1. The general equation is: \( a_n = (\alpha_1 + \alpha_2 n)1^n \)

The answer to the original question is:
\[ (\alpha_1 + \alpha_2 n + \alpha_3 n^3)(-1)^n + (\alpha_4 + \alpha_5 n)(2)^n + (\alpha_6 + \alpha_7 n)(5)^n + \alpha_8 7^n \]

7.5 #10 Find the number of positive integers not exceeding 100 that are not divisible by 5 or 7.

\[ 100 - \left\lfloor \frac{100}{5} \right\rfloor - \left\lfloor \frac{100}{7} \right\rfloor + \left\lfloor \frac{100}{35} \right\rfloor = 100 - 20 - 14 + 2 = 68 \]

7.6 #4 Find number of solutions to equation \( x_1 + x_2 + x_3 + x_4 = 17 \) where \( x_i, i = 1, 2, 3, \ldots \) are nonnegative integers such that \( x_1 \leq 3, x_2 \leq 4, x_3 \leq 5, x_4 \leq 8 \).

You find the answer to this one by finding out the number of ways four positive numbers can add up to 17. That is \( 20 \text{ C } 3 \). Then you figure out how many numbers must be
excluded. I’ll leave that as an exercise because it is extremely tedious and time consuming. Example 1 in sections 7.6 details how to solve the problem. I believe the final answer is 20.